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Notes: Equilibria and Optima

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What makes this intl? more than one agent = allocation problems
typically more than one good, too

Welfare theorems

First theorem: A competitive equilibrium allocation is Pareto optimal.

Second theorem: A Pareto optimum corresponds to a competitive equilibrium for some initial distribution of resources.

Where we're headed: find solutions based on optima
Warning: you'll think we're wasting out time, but we're not.

Exchange economies

[Board: 3 cols, phys env, ce, po]

Physical environment

- list of commodities ("commodity space," a vector x indexed by j)
- list of agents, their prefs (U_i) and endowments (y_i)

A *competitive equilibrium* consists of allocations $\{x_i\}$ (one vector for each agent i) and prices p satisfying:

(a) (agents maximize) Given prices p, each x_i maximizes U_i subject to the budget constraint $\sum_j p_j x_{ij} \leq \sum_j p_j y_{ij}$.

(b) (markets clear) $\sum_i x_i \leq \sum \sum_i y_i$.

Typical computational method: (a) do the max to find demand functions $x_i(p)$ then (b) solve $x(p) = \sum_i x_i(p) = 0$ for p. Fine point: x homogeneous of degree zero and satisfies Walras law ... so put on simplex (or other restriction).

We say an allocation $\{x_i\}$ is *feasible* if it satisfies the resource constraints, $\sum_i x_{ij} \leq \sum_i y_{ij}$, one for each commodity j. A feasible allocation is *Pareto optimal* if no other feasible allocation is preferred by one agent and no worse for others.

Not so user friendly. If each U_i is increasing, an optimum is the solution to a problem of the form:

$$\max_{\{x_i\}} U_1(x_1)$$

subject to

$$\begin{aligned} \sum_i x_i &\leq \sum_i y_i \\ U_i(x_i) &\geq \overline{U}_i \quad \text{for all} \quad j > 1. \end{aligned}$$

Even better, if each U_i is strictly concave, a Pareto optimum is the solution to the problem:

$$\max_{\{x_i\}} \quad \sum_i \theta_i U_i(x_i)$$

subject to

$$\sum_i x_i \leq \sum_i y_i$$

for some choice of "welfare weights" θ_i.

This leads to the so-called Negishi algorithm for finding a Pareto optimum: do the max for given weights θ_i using Lagrange multipliers p on the resource constraints. If we want to find the competitive equilibrium associated with this allocation, as in the Second Welfare Theorem, use p for the price vector.

Mantel developed this idea further into an algorithm for computing competitive equilibria. In what ways is the solution to the Negishi algorithm not a comp eq? Answer: doesn't satisfy budget constraints. The allocation associated with particular weights θ might be expressed $\{x_i(\theta)\}$. Define the savings vector (budget constraint deviations) by with components: $s_i(\theta) = \sum_j p_j(y_{ij} - x_{ij})$. Mantel suggested we find a competitive eq by solving: $s(\theta) = 0$ for the appropriate weights θ. In economies with many goods but few people, this is sometimes easier (lower dimension). Fine point: s homo of deg zero and sum to zero, so put θ on simplex.

For later: clarify the duality connecting equilibria and optima.

Example. Two agents, two goods, log utility. Preferences are the same for each: $U(a, b) = \alpha \log a + (1 - \alpha) \log b$. Endowments: Agent 1 has y_1 units of a, agent 2 has y_2 units of b. PO solves:

$$\begin{aligned} \max_{\{a_i, b_i\}} \mathcal{L} &= \theta_1[\alpha \log a_1 + (1 - \alpha) \log b_1] + \theta_2[\alpha \log a_2 + (1 - \alpha) \log b_2] \\ &\quad + p_1[y_1 - a_1 - a_2] + p_2[y_2 - b_1 - b_2]. \end{aligned}$$

[Comment: think about sub and superscripts...] FOCs:

$$\begin{aligned} a_i : \quad \theta_i \alpha / a_i &= p_1 \\ b_i : \quad \theta_i (1 - \alpha) / b_i &= p_2. \end{aligned}$$

Plug into resource constraints to find prices: $p_1 = \alpha/y_1$ and $p_2 = (1-\alpha)/y_2$. Use prices to find allocations: $a_i = \theta_i y_1$ etc.

What are budget constraints? For agent 1: $s_1(\theta) = p_1 y_1 - (p_1 a_1 + p_2 b_1) = \alpha - \theta_1$. CE is therefore: $\theta_1 = \alpha$.

Discussion. Consumption? Same share of each good. Relative prices? $q = p_2/p_1 = [(1-\alpha)/\alpha](y_1/y_2)$. Depends on supply. Symmetric? No, depends on α.

Homothetic preferences

Define... Monotonic function of HD1 function. In our example: HD1 function is $a^\alpha b^{1-\alpha}$ and monotone function is log. Useful for growth economies...

If agents have identical homothetic prefs, prices do not depend on distribution...

Example. Make the log weights different.

Production

Changes the resource constraints, but otherwise the same idea. We'll see how it works later on.

CE. add condition: firms max profits (and agents own firms).

PO. modify resource constraint.